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Abstract

This paper analyzes how the principal induces the dynamic incentives among agents through *both the monetary and non-monetary incentive schemes.* The principal commits herself to the incentive scheme at the beginning, intervenes in the competition among agents at the interim stage, and assign the production share in the final period. By doing so, an asymmetric equilibrium favorable for the winner is generated in the subsequent subgame, which in tern creates *the tournament effects through the discrete prize in the first period,* in addition to the marginal strategic effect. we characterize the optimal solution of the overall problem (a Kuhn-Tucker problem with the global incentive constraints)of the principal ,and investigate some conditions, under which the principal wants to induce the ex post competition among agents actively, under which she doesn't induce it and restrain the procurement from the loser, and under which she chooses not to intervene in the form of 'allotment,' even with the observability of the progress on the way among agents.

These solutions imply that the principal *endogenously chooses the mode of competition,* depending upon the exogenous conditions. This kind of intervention mechanism not only contributes to the growth of firms, but induces it *less costly.* It explains the efficiency of Japanese subcontracting systems, and also gives an important hint on the growth (catch up) of East Asian Countries.

Key words. Controlled Competition ,tournaments, renegotiation, technology transfer, allotment, Kuhn - Tucker problem ,slack, relaxing or tightening the incentive constraints.

Journal of Economic literature Classification Number D43, L14

1. Introduction

The aim of this paper is to investigate how the principal induces and controls the incentives of the two agents in the dynamic competition in organizations, using several instruments ,but under several constraints ,from the viewpoint of his or her private payoff.

This type of controlling dynamic competition in organizations is often called 'Controlled Competition' .It is a key concept for explaining the growth and development of organizations .

At first, let us present the two representative examples of such type of controlled competition, observed in the real world.

In the U.S defense procurement, the buyer (government or D.O.D) procures some units of a particular (perfectly divisible) item. Two firms (developers) could supply these units and compete with each other, seeking for a larger order from the government. This is known as 'Second Sourcing' (For a survey, See Anton and Yao (1990)).

Second, also in Japanese subcontract-systems, the buyer (the assembler) employs the two part suppliers (sub-contractors), and let them compete with each other, early in the development stages. This is commonly observed in the development activities of the parts of "the drawings approved", called the "Design-in". Itami (1986,88) points out that this type of competition promotes the accumulation of the relation specific skill, and contributes to the growth and the competitive advantage of the Japanese auto industries. As for these phenomena, we can understand that the principal (buyer) maximizes his or her profit achieved on the equilibrium path, through controlling the dynamic incentives among two agents by using the incentive schemes. Then, we need to consider not only the *monetary* incentive scheme, such as the monetary payment (bonus), but also the *non-monetary* incentive scheme, such as the allotment (quota), because the principal can increase her profit by reducing the cost for implementing the high level of incentives, with the help of the non-monetary incentive schemes.

In the 'controlled 'competition, the principal is faced with several incentive constraints. For example, he or she will have to satisfy the ex-ante and ex-post global incentive constraints (individually rational constraints),in order to induce the agents (the winner and especially the loser in the interim stage) to continue participating in the competition ,in each of two periods. This may constrain the objective of the principal ,and so he or she will have to contrive some instruments relaxing the constraints in order to attain the higher equilibrium payoffs. Or, when the technology transfer occurs in the interim stage in equilibrium, how does it affect upon the dynamic competition and the attainable payoffs through both inducing the ex post incentive and changing the tightness of the incentive constraints? These constraints may affect upon the nature of the ex-ante contracts ,and as a result, the mode of competition endogenously chosen may be changed.

As for a review of the literature, this paper can be placed along the line of the problems of designing the optimal contracts under several incentive constraints. For example, Itoh (1991) investigates the contract design by the principal in a multi-agents model from the viewpoint of whether and when she should induce each agent to provide the helping efforts for his coworkers, in addition to the own efforts. Kofman and Lawaree (1993) considers the optimal contract design under the possibility of side-contracting among several subgroups in a three-tier hierarchy. Our paper rather investigates the contract design of how the principal should choose the mode of dynamic competition among the agents ,under exogenous conditions. One focus is the interaction between the ex-ante and the ex-post global incentive constraints through two incentive instruments..

The solution of the paper corresponds to the endogenous determination of the market structure, which can be interpreted also in the view of the theory of the firm . The elimination tournament corresponds to the strategy of the principal of the concentration of the allocation of control right over the ex-post actions in *one agent* ,and the non-elimination tournament corresponds to the one of giving control right to two agents, letting them compete . As an interpretation in the view of the interaction between the *information structures* and the induced incentives , we can state that the principal may observe the *signal* on the progress on the way ,i.e., the first period capital accumulation outcome and determine the interim rank ,with the differential allotment pair ,if and only if that is specified in the initial contract. Also, the observability on the first period outcome among the agents generates *the marginal strategic effect* and reduces the incentive cost , though our paper also incorporates the allotment scheme of the principal ,generating *the discrete tournament effect* .[1]

This paper is organized as follows. In section 2, we present the two-stage procurement contract model. In section 3.1 through 3.3, we solve the model by the backward induction. The solution of the

[1] In the world of incomplete contracts, by inherent multiplicity of equilibria, information revelation and incentive compatible equilibrium selection, a tournament can be endogenously generated to strengthen the investment incentive in the relation specific skill. As for this ,see Konishi, Okuno-Fujiwara and Suzuki (1996).

overall problem corresponds to the situation where the principal endogenously chooses the mode of dynamic competition among the agents, depending upon the exogenous parameters. In section 4, we present the several theoretical extensions informally. In section 4, we conclude the paper.

2. The Model

Now, we consider the two sets of risk neutral players. We call them the principal and the agents. The setting of the model is based upon the model of transactional relationship of Grossman-Hart (1986). For example, we can image the Japanese parts transactional relationship, but the model is also related to the "second sourcing" model of the U.S defense procurement(1). The former of the two sets of the players is a buyer, and the latter is the set of potential sellers (suppliers), where the sellers (suppliers) supply the parts needed for the production of the final good (e.g. the automobile) to the buyer (assembler). The principal (the buyer or assembler) exercises an discretion over the organization structure and the suppliers can invest in the accumulation of the relation specific skill. Further, these investments are specific for the buyer. We assume that the principal can employ the two agents over time. (Of course, both parties have a decision move to stop the relationship on the way.)

Figure 1 around here

As you see from the figure 1, in the basic model, there exist two periods before the production and the sales occur. The accumulated capital stock remains to be unknown until the end of each period. That of agent i at the end of the second period is represented by the following stochastic variables.

$$\widetilde{K}_{3i} = K_{i2} + e_i + \varepsilon_i \qquad i=1,2 \tag{1}$$

Here, K_{i2} is the modified capital stock of agent i, after the transfer (spillover) of technology and knowledge between the end of the first period and the start of the second period. As is known from this

(1) See, Anton and Yao (1989) and Riordan and Sappington (1989). Laffont and Tirole (1993) analyzes the problems of incentives in procurement and regulation mainly in the framework of (static and dynamic) adverse selection.

equation, the final capital stock $\widetilde{K}_{i3}$ consists of the sum of the start of second period capital K_{i2}, the investment e_i and a noise ε_i, $i=1,2$, $i \neq j$. Here, the expectation of ε_i, $E(\varepsilon_i)=0$. and the variance of ε_i, $E(\varepsilon_i^2)=\sigma^2$. ε_i is independent of any other variable, including other ε_j. The principal and the agents know the distribution of ε. The objectives of the principal and the agents are to maximize their individual expected profits. The timing of a sequence of events is summarized as follows. First, the principal proposes an initial contract (incentive scheme) intended for both agents. The contents are two-folds. One of them is the division of production volume (ordered quantity) of the final period among the agents, according to <u>the interim rank</u>. The other is the monetary (down) payment, depending upon <u>the final rank</u> of the capital accumulation competition. In other words, the former is according to the relative performance (rank order) of the first period capital accumulation competition and the latter is to the final rank based upon the outcome of the second period competition . The suppliers are identical in the capital level *ex ante*, and they invest in the capital accumulation simultaneously and independently, given the production allotment scheme and the rule for the monetary payment in the second period, preannounced by the principal. The relative position of the first period competition determines the interim rank, and the relative position of the final capital accumulation levels, represented by (1), determines the final rank. The rule appoints that if an agent becomes a final winner, he or she is given an additive monetary prize. In addition, the principal can choose the way how he or she implements these instruments, that is, uses them based on the relative performance of what stage. After observing this rule, the agents accept or reject it[(2)] . The suppliers called the agents invest in the capital accumulation. These investments are unverifiable at the court, so, non-contractible. The idea of incorporating the investment in two periods before the production and the sales reflects the situation of continuous investments and updating the quality of product. This may be considered as the modeling of the unique Japanese transactional form where the skill or asset of technological know-how and the production skills are continuously generated and are improved and updated. This setting at the final stage given the accumulated capital configurations, if both sides desire the trade, the traded goods generate the value of (1) per unit production. We assume that the unit production (and sales) cost is zero. Hence, the gross joint surplus of trade is the final accumulated quality represented by (1). The surplus will go away when

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[(2)] The agents are allowed to quit the relationship (and to obtain its outside option value) at any time. Let $X_t=1$ if the agent accepts the incentive scheme at date t, and let $X_t=0$ if it quits, at date t.

the trade does not occur between them. The division of surplus is decided by the bargaining between them.

3. The Solution of the Model.

Hereafter, we solve this three stage game including a two stage tournament in a backward inductive fashion. First, in the section 3.1, we analyze the second period competition *(ex post competition after the first ranking was established.)*

3.1 The Second Period

According to the following characteristic of the "Japanese Style" competition form, that is, through the transfer of technology and knowledge, which can be interpreted as the "communication" after the ranking at the end of the first period, we assume that the difference between two levels of the technology of suppliers (developers) shrinks. That is to say, even if they reached the different capital stock configurations, the learning including the knowledge transfer or the R&D spillover occurs. When the end of first period stocks are $(\widetilde{K}_{i1}, \widetilde{K}_{j1})$, $i \neq j$, the start of the second period stock of agent i is, $K_{i2}(\widetilde{K}_{i1}, \widetilde{K}_{j1}) = \overline{K} + (\widetilde{K}_{i1} - \overline{K}) + t \cdot (\widetilde{K}_{j1} - \overline{K})$. Here, t is the real number, satisfying $0 \leq t \leq 1$, so the agent can learn the rate t of the skill accumulation of his competitor. For example, in Japan, the various institutions (Government councils and the private research circles etc.) are formed and, at the same time, the private networks for the exchange of information are utilized. It is well known that in auto/parts industries, too, the essence of the drawings contrived by one supplier is transferred to the rival supplier. This is especially often observed in "the drawings approved" and/or the development activities called "Design-in". Thus, we formulate the characteristic of the "Japanese Style" competition as the following assumption 1. Of course ,this is observed also in the U.S defense procurement systems.

Assumption 1 : The Linear Transfer Technology

The principal can offer the opportunity for transferring the knowledge and technology. By that, the difference of capital stocks decreases, and the start of second period capital stock is written as follows, given the end of first period stocks $(\widetilde{K}_{i1}, \widetilde{K}_{j1})$, $i \neq j$.

$$K_{i2}(\widetilde{K}_{i1},\ \widetilde{K}_{j1}) = \overline{K} + (\widetilde{K}_{i1} - \overline{K}) + t \cdot (\widetilde{K}_{j1} - \overline{K}) \text{ for } i \neq j,\ i=1,\ 2, \text{ and } 0 \leq t \leq 1$$

This assumption abstracts most simply the situation that over time, the principal intervenes into the process of competition among the few, fixed members, and he transfers the specific knowledge and/or technology to each other, for example, through the supplier associations in the automobile industry. Also in the U.S defense procurement competition, known as *second sourcing*, the developer (leader)'s technology and knowledge is transferred to the second source (the follower), who is then allowed to take over the production through bids against the developer.

This specification captures some essential ideas. The first point is the technological dependence among competitors. The second point is that the capital accumulation is effective conditional upon the amount of accumulation $(\widetilde{K}_{i1} - \overline{K})$ $i \neq j,\ i=1,2$. Lastly, the technology transfer may accompany the cost of adaptation and learning, which is formulated as $0 \leq t \leq 1$.

Agents know this learning process (linear transfer technology) and make investments in the first period, expecting this process (and considering the effect upon the second stage competition). The winner of the first period competition is given the favorable allotment in the production cartel at the end of next period, as a reward both for the victory and for transferring his knowledge capital. This assumption implies that after the first period, the principal gives an interim rank on the two upstream firms and then shrinks the difference between them by the amount of t times of it. That is, the difference of their capital stocks leads to $(1-t)\cdot(\widetilde{K}_{i1} - \widetilde{K}_{j1})$. Due to the linearity of technology transfer, the analysis becomes simpler without fundamental changes. Later, we investigate the effect of this knowledge (technology) transfer upon the equilibrium, and the solution of the model[(3)].

Assumption 2 : *At the time of sharing production gains, the principal and each agent split the gain from trade among them, according to the Nash bargaining. The bargaining power* α *is then, exogenously given in our model.*

One focus of this paper is how the dynamic competition among the agents is changed by the

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[(3)] Readers shouldn't consider this assumption as unimportant. We give the new insight on this transfer, different from the existing literature (Riordan-Sappington 1989).

strategy (φ and W) of the principal. The bargaining power $\alpha (0 < \alpha < 1)$ of each agent exerts various roles in the comparative statics in the model, which is not endogenously generated .Now, let us proceed to the detailed analysis.

3.1.1 The second period tournaments with production allotments.

At the start of the second-period, there exist two agents who are assigned different ranks and the ordered quantity based upon the outcome of the first period competition. We call them the winner and the loser, respectively. In this interim stage, the agents are not symmetric any more, different from the beginning of the first period. It is because they start their investments given the different production allotments. Nonetheless, due to the assumption 1 (spillover (transfer) of technology and knowledge), both agents have approached closely to each other in their skills (modified accumulated capital stocks that are observable among them). Under this situation, they solve the following problem simultaneously, given the modified capital stocks, K_{12} and K_{22}. Let V_{2w} and V_{2L} be the value functions for the winner and the loser, in the second period.

$$V_{2W}(a, Q, \Delta K; \varphi, W) = \max_{e_W} \underset{\varepsilon}{E}\left\{[a\tilde{K}_{2W}] \cdot [\varphi Q] + \Phi(\Delta K + \Delta e) \cdot W - C(e_W)\right\} \tag{2}$$

$$V_{2L}(a, Q, \Delta K; 1-\varphi, W) = \max_{e_L} \underset{\varepsilon}{E}\left\{[a\tilde{K}_{2L}] \cdot [(1-\varphi)Q] + \left\{1 - \Phi(\Delta K + \Delta e)\right\} \cdot W - C(e_L)\right\} \tag{3}$$

These mathematical representations are the problems that both the winner and the loser face in the second- period. Here, we postpone the analysis of the *global* incentive constraints of both agents later. The meaning of equations (2) and (3) are as follows.

①Agent i is assigned the production volume at the final production stage (cartel), according to the rank order in the capital accumulation competition in the first period, as follows.
The allotment scheme is[4]

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[4] Matsui (1989) investigated the cartel rule for improving the consumer surplus *generally*. This paper doesn't have such concern, but investigate the optimal contract under the local and global incentive constraints in each of two periods explicitly. The principal chooses the optimal rule from among the set of credible contracts satisfying these constraints.

$$q_i = \begin{cases} \varphi \cdot Q & if \quad \widetilde{K}_{i1} > \widetilde{K}_{j1} \\ (1-\varphi)\cdot Q & if \quad \widetilde{K}_{i1} < \widetilde{K}_{j1} \end{cases}$$

where q_i is the instructed ordered quantity to agent i. φ is the production share assigned to the winner of the first period, and it satisfies $\frac{1}{2} \le \varphi \le 1$.

That is, the principal usually increases the ordered quantity for the winner in the first period. We assume the inelastic demand curve, and the demand for the final goods (e.g. autos) that the principal produces is constant at the level of Q.

② $\widetilde{K}_{2W}$ and $\widetilde{K}_{2L}$ are the final qualities per unit the goods accumulated by the winner and the loser, until the production and sales stages. These correspond to the valuations for the users or buyers. By assuming zero production cost, these correspond to the gross trading value.

③ α is the bargaining power of the agent, exogenously given. This is the distribution share to him or her of the trade gain. Hence, $\widetilde{K}_{2W}$ and $\widetilde{K}_{2L}$ can be, respectively, viewed as the consumer price of the final good with each characteristics or type (for example, imagine each type of cars). $\widetilde{P}_W = \alpha \cdot \widetilde{K}_{2W}$ and $\widetilde{P}_L = \alpha \cdot \widetilde{K}_{2L}$ means the unit revenue (input prices) of the (final) winner and the(final)loser .respectively(6) .

④W is the (monetary) prize given to the final winner, who has the larger capital $\widetilde{K}_{2W}$ than $\widetilde{K}_{2L}$. We investigate later as to why the principal should set the monetary bonus W at this stage, based upon the second period relative performance (rank order)(6) .

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(6) The way that the consumer price of the good will be determined is not explicitly dealt with. But, it is easy to contrive a mechanism which implements the price in the text in equilibrium. Also, it does not change the essence of what I will send to readers qualitatively, even if we consider another game which implements another part price (price of intermediate goods), for example, the bargaining between firm and consumers.

(6) Since W is a lumpy prize, the structure of the second period competition is an *asymmetric tournament*, given a different allotment pair to the two agents. This is a different technical point from Konishi etal (1996).

⑤ $\Phi\ (\Delta K+\Delta\ e)$ is the winning probability in the second period for the winner in the former period, when the winner and the loser make the investments at the level of e_W and e_L, given ΔK (the difference of the capital stocks between them). This is a function of $\Delta K+\Delta\ e$, where $\Delta\ e = e_W - e_L$

⑥C is the cost function of investment in the second period, with $C'>0$, $C''>0$ and $C'''\geq 0$.

Now, the rewinning probability of the winner in the second is given by the following formula.

$$\begin{aligned}
&\Phi(\Delta K+\Delta\ e) := Prob\ (\widetilde{K}_{2W} \geq \widetilde{K}_{2L}) \\
&= Prob\ (\Delta K + e_W - e_L > \varepsilon_L - \varepsilon_W) \\
&= \Phi(\Delta K + e_W - e_L)
\end{aligned}$$

Here, Φ is the distribution function of the random variable $\varepsilon_L - \varepsilon_W$, and we denote the density function by the small letter ϕ. We assume that ϕ is decreasing function for positive values, i.e. $\phi'(\chi) < 0$ for $\chi > 0$. ϕ has a finite support $[-\bar{\varepsilon}, \bar{\varepsilon}]$, and is symmetric within that range. i.e., $\phi(\chi) = \phi(-\chi)$ for $\forall\chi \in [-\bar{\varepsilon}, \bar{\varepsilon}]$.

Now, the first order conditions for the second period optimization problems of the agents are,

$$\alpha \cdot [\varphi Q] + \frac{\partial\Phi(\Delta K + e_W - e_L)}{\partial e_W} \cdot W - C''(e_W) = 0 \qquad (5)$$

$$\alpha \cdot [(1-\varphi)\cdot Q] - \frac{\partial\Phi(\Delta K + e_W - e_L)}{\partial e_L} \cdot W - C''(e_L) = 0 \qquad (6)$$

In other words, the following two simultaneous equations represent the Nash equilibrium in the second period under the assigned production allotments.

$$\alpha \cdot \varphi Q + \phi(\Delta K + e_W - e_L) \cdot W = C'(e_W) \qquad (7)$$

$$\alpha \cdot (1-\varphi)Q + \phi(\Delta K + e_W - e_L) \cdot W = C'(e_L) \qquad (8)$$

The first terms of the formula (7) and (8) show the marginal revenue of the (ex post) capital accumulation, given the assigned ordered quantity. The second terms show the marginal value products of the second period investments, through the marginal improvement of the probability of getting the monetary payment (subsidy) W. To ensure the existence of an optimum in the second period problem, we assume that the following second order conditions are satisfied.

$$\phi'(\Delta K + e_W - e_L)\cdot W - C''(e_W) < 0$$
$$-\phi'(\Delta K + e_W - e_L)\cdot W - C''(e_L) < 0$$

[Proposition 1]

In the equilibrium of the second period investment competition, the leading firm (the winner in the first period) never invests less than the follower (the loser in the first period).

[Proof]

In order to compare the equilibrium incentives of both agents, we combine equations (7) and (8), and get $C'(e_W) - C'(e_L) = (2\varphi - 1)\cdot \alpha Q$ (9)

Noting that $\alpha > 0$, $\varphi \geq \frac{1}{2}$, and C'(e) is increasing in e , it follows that

$$e_W^* \geq e_L^* \quad (10)$$

This proposition implies that the leader always makes more efforts than the follower, because the principal assigns the favorable share (ordered quantity) $\varphi > \frac{1}{2}$ to the first period winner, which brings about the difference of $(2\varphi-1)\,\alpha Q$ between the marginal productivities that results in the asymmetric equilibrium in the second period. (when $\varphi = \frac{1}{2}$, it follows that $e_W^* = e_L^*$). Further, we get the proposition on the comparative statics of the Nash Equilibrium in the second period.

[Proposition 2]

An increase in the size of the prize W induces the increase in both agents' incentives, but the difference between them shrinks. In other words, the marginal incentive of investment is larger for the loser than for the winner.

That is,

$$\frac{\partial e_W^*}{\partial W} > 0, \frac{\partial e_L^*}{\partial W} > 0, \frac{\partial (e_W^* - e_L^*)}{\partial W} \le 0$$

Proof.

Differentiating equations (F.O.Cs) (7) and (8) and setting $d\varphi = d\alpha = 0$, we obtain the following matrix representation.

$$\begin{bmatrix} \phi' \cdot W - C''(e_W) & -\phi' \cdot W \\ \phi' \cdot W & -\phi' W - C''(e_L) \end{bmatrix} \begin{bmatrix} \frac{\partial e_W^*}{\partial W} \\ \frac{\partial e_L^*}{\partial W} \end{bmatrix} = \begin{bmatrix} -\phi \\ -\phi \end{bmatrix}$$

Here, we check whether the stability condition is satisfied. Let the Hessian determinant be | D |. | D | $=(\phi' \cdot W - C''(e_W)) \cdot (-\phi' \cdot W - C''(e_L)) + (\phi' \cdot W)^2 > 0$ The positive sign was derived from the second order conditions (S.O.Cs) and the conditions on Φ and C.

Solving the matrix systems by using the Cramer's Rule, we can obtain

$$\frac{\partial e_W^*}{\partial W} = \frac{|D_1|}{|D|} = \frac{\phi \cdot C''(e_L^*)}{|D|} > 0$$
$$\frac{\partial e_L^*}{\partial W} = \frac{|D_2|}{|D|} = \frac{\phi \cdot C''(e_W^*)}{|D|} > 0$$

Considering the difference in equilibrium incentives, we get

$$\frac{\partial (e_W^* - e_L^*)}{\partial W} = \frac{|D_1| - |D_2|}{|D|} = \frac{\phi \cdot [C''(e_L^*) - C''(e_W^*)]}{|D|} \le 0$$

This result is obtained from $C''' \ge 0$, and $e_W^* \ge e_L^*$ $/D/ > 0$.

QED

In order to understand this result intuitively, it is necessary to remember that the prize (subsidy) in the second period is given to the final winner, based upon the final rank irrespective of the interim rank of the capital accumulation competition. From the F.O.Cs (7) and (8), the increase of the prize W implies the increase of the marginal value products for both agents. Thus, it has a positive effect upon the second-period investments of both agents. By the way, the marginal revenue for each agent of increasing W is the same value, $\phi(\Delta K + e_W - e_L)$.

Noticing that the winner has more investments in equilibrium, $e_W^* > e_L^*$, the same marginal revenue induces more incentive from the loser, under the convexity of the cost function in effort incentive. We recognize from this fact that the increased subsidy in the second period shrinks the difference between the equilibrium incentives of two agents.

[Corollary 1]

We assume that $C(e) = \frac{1}{2}e^2$. In this case, using the F.O. Cs (7) and (8), the equilibrium investment levels are

$$e_W^*(\varphi, W; \Delta K, \alpha, Q) = \alpha \cdot \varphi Q + \phi^* \cdot W$$
$$e_L^*(\varphi, W; \Delta K, \alpha, Q) = \alpha \cdot (1-\varphi) Q + \phi^* \cdot W$$

where $\phi^* = \phi(\Delta K + e_W^* - e_L^*) = \phi(\Delta K + \alpha \cdot (2\varphi - 1) \cdot Q)$

Then, by the statements of formula (11),

$$\frac{\partial e_W^*}{\partial W} = \frac{\partial e_L^*}{\partial W} = \phi^* > 0, \frac{\partial (e_W^* - e_L^*)}{\partial W} = 0$$

This implication is that, the increased prize W of the second period has a positive effect upon the equilibrium investment of each agent, but in the quadratic cost function case, the size of the effort increase is the same, and so, the difference between effort incentives remains unchanged. On the other hand, as proposition 2 shows, in the case of $C(e) = \frac{R}{2}e^{\beta}$ where $\beta > 2$ and $R > 0$, $e_W^* - e_L^*$ decrease as W increases.

[Corollary 2]

As $t \rightarrow 1$, the investment levels increase in the competitive equilibrium in the second period.

Proof.

From F.O. Cs (7) and (8), the difference of investment levels remains the same, irrespective of t, given the other parameters α, Q and φ. As $t \rightarrow 0$, the modified difference $\Delta K = (1-t)\cdot(\tilde{K}_{11} - \tilde{K}_{21})$ increases, with decreasing the marginal productivity (probability density) $\phi(\Delta K + e_W - e_L)$. Then, the equilibrium incentives e_W^* and e_L^* decreases from F.O. Cs (7) and (8). Oppositely, both e_W^* and e_L^* increases as $t \rightarrow 1$ (technology transfer is progressed).

QED.

Let us further investigate the above results, in the viewpoint of *"strategic substitutability and complementarity"*. According to the ranking based upon the outcome of the first period competition, the difference is imposed between the shares in the production cartel at the end of second period, resulting in the difference of $(2\varphi-1)\cdot\alpha Q$ in the marginal productivities of investment in the second period. It is why the asymmetric equilibrium arises in the second period, and from (10), the winner exerts more efforts than the loser in equilibrium. Differentiating (7) and (8) representing the Nash equilibrium as to e_L and e_W, respectively, we obtain the following effect on the marginal profitability.

$$-\phi'(\Delta K + e_W^* - e_L^*) \cdot W > 0 \tag{12}$$

$$\phi'(\Delta K + e_W^* - e_L^*) \cdot W < 0 \tag{13}$$

Figure 2-1 around here

That is, in the neighborhood around the second period equilibrium, the investments are strategic complements for the winner, and strategic substitutes for the loser. This implies that these two agents react differently when $\frac{1}{2} < \varphi < 1$. The reaction function of the leading firm (the winner) is increasing in the follower's (the loser's) effort, whereas the reaction function of the trailing firm (follower) is decreasing in the leader's (the winner's) effort.

This figure suggests that the leading firm has more incentive to invest (*becomes aggressive*) when it faces the intense competition against the follower, whereas the trailing firm has more incentive to invest when the winner (rival) becomes *less aggressive.*

As a the comparative statics, we can obtain the effect of the increased φ upon the equilibrium incentives.

[Proposition 3]

The effect of the change of φ upon the Nash equilibrium in the second period is as follows.

$$\frac{\partial e_W^*}{\partial \varphi}?\quad \frac{\partial e_L^*}{\partial \varphi}<0\quad \frac{\partial(e_W^*-e_L^*)}{\partial \varphi}\geq 0\qquad \frac{\partial(e_W^*+e_L^*)}{\partial \varphi}<0$$

,and $$\frac{\partial[\varphi e_W^*+(1-\varphi)e_L^*]}{\partial \varphi}=(e_W^*-e_L^*)+\varphi\frac{\partial(e_W^*-e_L^*)}{\partial \varphi}+\left(\frac{\partial e_L^*}{\partial \varphi}\right)$$

We omit the proof because it is based upon the same procedure as the proposition 2. This implies that by the increase ofφ (the volume of allotment assigned to the winner), it discourages the loser largely after the implementation of it, thereby the difference between equilibrium incentives increases. This is a contrast with the results on the effect of W. The last term represents how the weighted average: $\varphi e_W^*+(1-\varphi) e_L^*$ of the equilibrium incentives are affected by the marginal change of φ. The sign is ambiguous because the first and second terms are positive, but the third term is negative. As for it, compare two figures 2-2 and 2-3.

Figure 2-2 and 2-3 around here

Summarizing the above analysis, the expected profits which both the winner and the lower obtain respectively in equilibrium are as follows.

$$V_{2W}^*=\alpha\cdot(K_{i2}+e_W^*)\cdot(\varphi Q)+\Phi^*\cdot W-C(e_W^*) \tag{14}$$
$$V_{2L}^*=\alpha\cdot(K_{i2}+e_L^*)\cdot(1-\varphi)Q+(1-\Phi^*)\cdot W-C(e_L^*) \tag{15}$$

,where $\Phi^*=\Phi(\Delta K+e_W^*-e_L^*)\geq\frac{1}{2}, K_{i2}=\overline{K}+(h_i+\varepsilon_i)+t\cdot(h_j+\varepsilon_j)$

and e_W^* and e_L^* is a function of ΔK, α, Q and φ, W.

Now, the difference between the equilibrium profits is,

$$\Delta V^*(\alpha, Q, K_{i2}; \varphi, W) = \alpha \cdot (2\varphi-1) \cdot Q \cdot K_{i2} + \{\alpha Q[\varphi\, e_W^* - (1-\varphi)\, e_L^*] + (2\Phi^*-1) \cdot W - [C(e_W^*) - C(e_L^*)]\} \qquad (16)$$

This is *the discrete prize* which positively induces the ex ante (first period) incentives from the agents when $\varphi > \frac{1}{2}$, and this prize establishes the *ex ante* competition. Let me summarize the mechanism which works so far in the model. The principal announces and commits herself to the incentive scheme, where the principal evaluates agents based upon the interim ranking (relative performance) of the outcome of the first period competition, and then changes the production share in the supply market favorable to the winner *discretely*. Thereby, when the two agents (the leader and the follower) compete for the monetary prize in the second period, there exists an asymmetric (pure strategy) equilibrium due to the difference of the marginal productivities, resulting in the difference between the equilibrium profits of two agents. This in turn works as a *carrot (prize)* which enhances the ex ante incentives of two agents. Therefore, when $\varphi > \frac{1}{2}$, the agents are faced with the tournament scheme with the prize of (16), where the reward is *discontinuously* based upon his performance see Figure 3. This is the essential logic of the analysis so far. The global incentive constraint in the second period (interim individually rationality constraint) will be investigated later.

Figure 3 around here

3.2 The first period

At the start of the 0-period, the principal is faced with the homogeneous agents with the capital stock level $\overline{K}$. She announces an organizational structure (incentive scheme), concretely, the rule for the production allotment in the final period and the rule of competition for the monetary prize. The former is related to whether the principal *endogenously* chooses the nonelimination tournaments *($\varphi \neq 1$)*, where the loser is not eliminated and given the one more chance in the second period, or the elimination tournament $(\varphi = 1)$, where the loser is made to drop out of the race in the second period, or the intertemporal monopoly by the single supplier.

After accepting the contract, two agents choose the capital investment levels of the first period, so as to maximize their own expected payoffs. Now, we suppose that the capital stock of agent i at the end of the first period is $\widetilde{K}_{i1} = \overline{K} + h_i + \varepsilon_i \quad i \neq j, i = 1,2$. $\overline{K}$ is the capital stock at the start point, observable among the principal and two agents. h_i is the investment level during the first period. ε_i is the uncertainty factor (the random shock in the first period). Next, we suppose that F(h) ,where $h = h_i - h_j$, is the winning probability of agent i in the first period capital accumulation competition. Then,

$$F = Prob(\widetilde{K}_{i1} > \widetilde{K}_{j1}) = Prob(h_i - h_j > \varepsilon_j - \varepsilon_i) = F(h_i - h_j) \tag{18}$$

ε_j-ε_i is the difference between the noise ε_j and ε_i, and thus F is the distribution function of the random variable ε_j-ε_i.

From the analysis in the former section, we see that two agents are faced with the following tournament scheme.

$$S_i(\widetilde{K}_{i1}, \widetilde{K}_{j1}) = \begin{cases} V_{2L} & if \quad \widetilde{K}_{i1} < \widetilde{K}_{j1} \\ V_{2L} + \Delta V & if \quad \widetilde{K}_{i1} > \widetilde{K}_{j1} \end{cases}$$

If the agent wins the race in the first period, he can obtain *the discrete prize* ΔV in addition to the base value V_{2L}. See Figure 3. The reward of agent i is based upon his absolute performance $\widetilde{K}_{i1}$ *discontinuously*. This is different from the relative performance scheme where the reward is based upon his absolute performance $\widetilde{K}_{i1}$ continuously. The important point of this scheme is the discontinuity, rather than the nonconcavity, because it generates the source for establishing the ex ante fierce competition between agents.

Two agents solve the following problems simultaneously and independently, given the rival's investment hj.

$$\max_{h_i} \delta \cdot \underset{\varepsilon}{E}\{F(h) \cdot V_{2W}^* + (1 - F(h)) \cdot V_{2L}^*\} - g(h_i) \qquad i \neq j, i = 1,2 \tag{19}$$

The fundamental equation of this problem is, given hj,

$$V_i(\overline{K},\overline{K},hj) = \max_{h_i} \delta \cdot \{V_{2L}{}^* + F(h)\cdot \Delta V^*\} - g(h_i) \qquad i \neq j, i = 1,2 \tag{20}$$

The first term $V_{2L}{}^*$ represents the value which he gets in the Nash equilibrium in the second period, when he loses in the first period competition, given $\Delta K, \varphi$ and W (the principal's two instruments). The second term F(h) • ΔV is the expected prize, which implies that with the probability of F(h), the agent can get this lumpy prize, which is endogenously generated as the equilibrium payoff difference in the second period through the incentive scheme φ and W. g(h_i) is the cost function of investments h_i in the relation specific skills, with $g'>0$, $g''\geq 0$. Now , we shall define $\overline{\upsilon}$ and $\underline{\upsilon}$ as follows.

$$\overline{\upsilon} := \overline{\upsilon}(\alpha,Q;\varphi,W) = (\alpha \cdot e_W{}^*)\cdot(\varphi Q) + \Phi^* \cdot W - C(e_W{}^*)$$

$$\underline{\upsilon} := \underline{\upsilon}(\alpha,Q;\varphi,W) = (\alpha \cdot e_L{}^*)\cdot(1-\varphi)Q + (1-\Phi^*)\cdot W - C(e_L{}^*)$$

where $\Phi^* := \Phi(\Delta K + e_W^* - e_L^*) \geq \frac{1}{2}$ (the equality is satisfied when $\varphi = \frac{1}{2}$ and $\Delta K = 0$.)

Solving the problem, we can obtain the F.O.C for agent 1.

$$\begin{aligned}\delta\cdot\Big\{&\alpha(1-\varphi)Q\cdot 1 + F(h_1-h_2)\big[(2\varphi-1)\cdot\alpha Q\cdot 1 + 2(1-t)\phi(\Delta K + e_W^* - e_L^*)\big]\\ &+\big[(2\varphi-1)\alpha Q K_{12} + (\overline{\upsilon}-\underline{\upsilon})\big]f(h_1-h_2)\Big\} = g'(h_1)\end{aligned} \tag{21}$$

Here, $K_{12} = \overline{K} + (h_1 + \varepsilon_1) + t\cdot(h_2 + \varepsilon_2)$ implies that the RHS (right hand side) consisting of initial stock $\overline{K}$ plus the capital (skill) accumulation $(h_1 + \varepsilon_1)$ by agent 1 plus the rate of t of the accumulation by the rival suppliers lead to the modified stock level K_{12} (LHS) at the start of period 2.

The equation (21) implicitly defines the reaction function of agent 1 in the first stage. The similar conditions can be obtained with respect to agent 2. Since both agents have identical skills at the beginning of the first period and so they are identical, in a symmetric equilibrium, the incentives are $h_1{}^*=h_2{}^*=h^*(\varphi, W;\ \alpha, Q, t, \delta)$.

In this case, the first order conditions are simplified as follows, which characterize the symmetric subgame perfect Nash equilibrium investment level.

$$\delta \cdot \{\alpha(1-\varphi)Q + F(0)\cdot[(2\varphi-1)\alpha Q + 2(1-t)\phi(e_W * - e_L *)\cdot W] + [(2\varphi-1)\alpha Q(\overline{K} + (1+t)h^*) + \overline{\upsilon} - \underline{\upsilon}]f(0)\} = g'(h^*) \tag{22}$$

The three terms in the blacket { }represent the following effects. The first term represents the marginal increase of the value V_{2L}*(formula (15)) that is expected to obtain in the second period Nash equilibrium, when he loses in the first period competition (race) through increasing the capital investment in the first period. The second term represents the marginal increase of the discrete prize: V_{2W}*-V_{2L}* itself, with the winning probability of $F(0) = \frac{1}{2}$ in equilibrium. The two terms of the blacket are , respectively, *the direct effect* and *the marginal strategic effect* .The direct effect means the marginal revenue from the increase of the ordered quantity with equal probability in equilibrium. The marginal strategic effect implies the strategic incentive for the agents to increase marginally the winning probability in the final period through increasing the difference of the first period capital accumulation. The third term is *the tournament effect* through marginal improvement of winning probability, given the equilibrium payoff difference, that is, the discrete prize. By summing up the two direct effects, the F.O.Cs (Local Incentive Constraints) are transformed as follows.

$$\delta \cdot \left\{\frac{1}{2}\alpha Q + (1-t)\phi(e_W * - e_L *)W + [(2\varphi-1)\alpha Q(\overline{K} + (1+t)h^*) + \overline{\upsilon} - \underline{\upsilon}]f(0)\right\} = g'(h^*) \tag{22'}$$

Further, in this two stage game, the global incentive constraint must be satisfied in the ex ante stage in the first period symmetric equilibrium. The agents can choose an alternative to deviate from the intense competition, becoming contented with the position of the loser in the tournaments, and so they can get the following intertemporal payoff.

$$U = \max_{h_l}\{\delta \cdot V_{2L} * - g(h_l)\} = \delta \cdot \left\{\alpha(1-\varphi)Q(\overline{K} + h_L + th^*) + \underline{\upsilon} - g(h_L)\right\} \tag{23}$$

where h_L is the maximand of the above formula. Each agent can secure at least the payoff of $\underline{\upsilon}$ by choosing the level of h_L when the rival may choose h*. Now, let us check the global incentive constraint or the ex ante individual rationality constraint in the first period.

$$\delta\cdot\frac{1}{2}\left[V_{2W}*(h^*,h^*)+V_{2L}*(h^*,h^*)\right]-g(h^*)\geq U \tag{24}$$

$$\Leftrightarrow \delta\cdot\frac{\alpha Q}{2}\left\{\overline{K}+(1+t)h^*+\left[\varphi e_W*+(1-\varphi)e_L*\right]\right\}+\frac{\delta}{2}\cdot W-\frac{\delta}{2}\left[C(e_W*)+C(e_L*)\right]-g(h^*)$$

$$\geq U=\delta\left\{\alpha(1-\varphi)Q(\overline{K}+h_L+th^*)+\underline{\upsilon}\right\}-g(h_L)$$

$$=\delta\left\{\alpha(1-\varphi)Q(\overline{K}+h_L+th^*+e_L*)+(1-\Phi(\Delta K+e_W*-e_L*))\cdot W-C(e_L*)\right\}-g(h^*)$$

Putting this inequality in order, we obtain

$$\delta\cdot\frac{\alpha Q}{2}\left[\overline{K}+(1+t)h^*\right]+\frac{\delta}{2}\left[\overline{\upsilon}+\underline{\upsilon}\right]-g(h^*)\geq\delta\alpha Q(1-\varphi)(\overline{K}+h_L+th^*)+\delta\cdot\underline{\upsilon}-g(h_L) \tag{25}$$

Hence, we obtain the following proposition 4 about the (ex ante) global incentive constraint.

[Proposition 4]: The Global Incentive Constraints in the first period.

There exists a symmetric equilibrium above the level of h_L in the first period only if

$$\delta\left\{\left[\frac{\alpha Q}{2}\right]\cdot\left(\overline{K}+(1+t)h^*\right)+\frac{1}{2}(\overline{\upsilon}-\underline{\upsilon})-\alpha Q(1-\varphi)\left(\overline{K}+h_L+th^*\right)\right\}\geq g(h^*)-g(h_L) \tag{26}$$

The first term in the blacket of the LHS of (26); $\left[\frac{\alpha Q}{2}\right]\cdot\left(\overline{K}+(1+t)h^*\right)$ represents the expected revenue obtained from the first period capital accumulation, because the agents can get the one-half of the total allotment Q in expectation, under which he is accumulating the total capital of $\overline{K}+(1+t)h^*$ in the first period per unit of the allotment and gets the share of α of it, due to the bargaining power. The second term in the blacket is the expected prize given the winning probability in equilibrium (the payoff spread $(\overline{\upsilon}-\underline{\upsilon})$ generated in the asymmetric equilibrium in the second period, due to the production allotment policy to the agents). The right hand side (LHS) is the extra cost when he or she chooses the equilibrium investment level h*, not the default level h_L, given the rival's investment behavior of choosing h*.

We can interpret this proposition 4 clearly, including the interpretation of the third term of the

LHS, in terms of the incentive constraint in the finitely repeated games [8]. In the LHS, the second term shows that the agents can obtain the prize of $\overline{\upsilon} - \underline{\upsilon}$ with probability $\frac{1}{2}$, if he or she plays the equilibrium level h*, when the opponent (rival) plays h*.

Nonetheless, if he deviates from h* to h_L, he loses with certainty, obtaining the allotment of $(1-\varphi)Q$, under which he gets the share of α of the total quality (gross gain from trade), consisting of initial stock $\overline{K}$, his investment h_L generated by the first period capital accumulation, and t times the rival's larger investment h*. If he had invested the equilibrium level of h*, he would have obtained the larger benefit from the own capital accumulation itself through getting the larger allotment, even with the probability of one-half.

The sum of these two terms is *the continuation loss* (or the penalty imposed upon the first period shirking (deviation) in terms of the second period payoff) of deviating from h* to h_L in the first period. On the other hand, the LHS represents the cost saving of the investment due to the deviation (Shirking or Cheating). This is *the deviation incentive*. Therefore, if the continuation loss is larger than the deviation incentive, then the investment level of h* can be supported as a subgame perfect equilibrium in the first period. Now, when the condition (26) is satisfied, we can compare the local (marginal) incentive in the first period on the equilibrium path with that of the case when only one agent is employed. In the case of one agent (bilateral monopoly), the agent is given the rate of α of the end of second-period capital stock $\widetilde{K}_2$ represented by (1), which is equivalent to the gross total value of trade. In other words, the agent considers his or her bargaining power equivalent of the capital stock $\widetilde{K}_2$, as the private revenue, when he or she decides the first and the second period investments. His objective function at the beginning of the first period can be described as

$$\delta \cdot \underset{\varepsilon}{E}\left[\alpha Q \cdot \widetilde{K}_2 - C(e)\right] - g(h) \tag{27}$$

He chooses the first period and the second period investment levels, so as to maximize the formula (27). The equilibrium investment level h^s and e^s satisfy the following F.O. Cs.

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[8] See, for example, Benoit, P and V, Krishna (1985).

$$\alpha Q = C'(e_s) \tag{28}$$

$$\delta \ \alpha Q = g'(h_s) \tag{29}$$

From these ,we can recognize the following facts. First, the agent underinvests from the view of the whole organization, because he can get only a small part α of the value added which he generates through his capital investment. This is so called *"Hold up problem"* (more generically, Free Rider Problem). Second, we find that the effort made in period 2 is greater than in period 1. This is an end effect in period 2. In the case where the two agents compete over two periods, the equilibrium incentive in the first period is characterized by the first order condition (22‘). Comparing the F.O.Cs in the two cases, first ,there exists the difference of the size of

$$\delta \cdot \alpha Q - \frac{1}{2}\delta\alpha Q = \frac{1}{2}\delta\alpha Q \tag{30}$$

as for the direct effect of capital accumulation. Next, let us consider the marginal strategic effect ; $(1-t)\phi(e_W^* - e_L^*)W$ .It implies the incentive for the agents to increase marginally the winning probability in the final period through increasing marginally the difference of the end of the first period capital stocks against the rival agent. This effect does not exist in the one supplier case (monopoly case). The tournament effect has the following implication. In the case of the competition for the allotment in the production "cartel" (that is, in the main model), the allotment in the final stage (production and sales stage) is based upon the ranking (the rank order) of the capital accumulation in the first period. Under such an allotment mechanism, when the agent exerts the larger capital accumulation and win the race, he obtains a relatively large allotment that implies the favorable position for the ex post competition in the second period, whereby he can obtain the additional expected profit in equilibrium. He or she expects this prize rationally, and compete with his rival "head-to-head" so as to increase the probability of getting it. The indirect incentive effect resulting from this behavior is *the "Tournament" effect.* We compare the relative size of these two effects, obtaining the following proposition on the ex ante marginal (local) incentive in equilibrium.

[Proposition 5]

Comparison between marginal incentives :

Suppose that there exists a unique symmetric equilibrium in the first period, i.e., the inequality (26) in *the proposition 4 as well as the local conditions (22) is satisfied. Then, the equilibrium incentive h* is larger than h^s (the investment level under the one supplier case) if and only if*

$$(1-t)\phi(e_W{}^*-e_L{}^*)\cdot W+f(0)\{\alpha Q[2\varphi-1][\overline{K}+(1+t)h_s]+(\overline{\upsilon}-\underline{\upsilon})\}\geq\frac{1}{2}\alpha Q \tag{31}$$

The important element is *the tournament effect* as the second term of the LHS ,which consists of the two components. One is the term f(0), which implies the marginal improvement of winning probability at the symmetric equilibrium. The other is the size of prize (payoff difference) generated in the asymmetric equilibrium, corresponding to the terms in the blacket { }. The first term in the blacket, $\alpha Q[2\varphi-1][\overline{K}+(1+t)h_s]$ is the difference in the revenue evaluated at h_s, resulting from the part of the first period capital accumulation, based upon the difference of the rank-order in the first period competition. $[2\varphi\text{-}1]Q$ represents the difference in the assigned allotments. Due to it ,the winner can get the more revenue, even with the equal equilibrium incentives. The second term, $\overline{\upsilon}-\underline{\upsilon}$ represents the payoff difference in the second period between the winner and the loser. Of course, the marginal strategic effect;$(1-t)\phi(e_W^*-e_L^*)W$ is positive for $0<t<1$. On the other hand, the RHS is the direct effect (30), which implies that the competition has a negative incentive effect from this point of view. Here we can recognize the following facts. First, as f(0) is larger, that is, as the support of uncertainty at the end of the first period becomes smaller, this inequality tends to be satisfied. In addition, as is larger the discrete prize in equilibrium, the investment over the hold up level defined by (29) tends to be induced. Nonetheless, note that it is another problem whether that principal really has an incentive to induce the above incentive. We analyze this principal's optimization problem in the next section.

Figure 4 around here

3.3 The 0 period : The principal's Problem

Last, we focus on the behavior of the principal. This is the problem on the optimal design of contract for her to maximize her private profit. At the beginning of the 0 period, the principal chooses both φ (the production share assigned to the first period winner) and W (the monetary rewards given to the final winner), and commits herself to the two strategies. Her objective function is,

$$\pi^*(\varphi, W) = \delta \cdot (1-\alpha) \cdot Q\{(\overline{K} + (1+t)h^*) + [\varphi \cdot e_W{}^* + (1-\varphi) \cdot e_L{}^*]\} - W \qquad (32)$$

This formula implies the share of the principal of the total value generated by the induced incentives, minus her fixed cost (monetary prize). After all, the principal's problem is to maximize the expected payoff $\pi^*(\varphi, W)$ defined as the function of both φ and W, subject to the inequality constraints $\frac{1}{2} \leq \varphi \leq 1$ and $W \geq 0$, the global incentive constraints (26) and $V_L{}^* \geq 0$, corresponding to the ex ante and interim (ex post) participation constraints of agents. Hence, the problem is formulated as follows. The principal chooses the optimal (φ, W) to maximize his expected payoff at period 0 from the set of credible contracts which satisfies these incentive constraints.

[Problem]

Max $\pi(\varphi, W) := \delta(1-\alpha)Q[\varphi \cdot \widetilde{K}_{2W} + (1-\varphi) \cdot \widetilde{K}_{2L}] - W$

$\{\varphi, W\}$

s.t · (26) ····· the global incentive constraint of the agents in the perfect equilibrium in the first period.

· $V_{2L}^* \geq 0$ ···the participation constraint of the loser to the ex post competition.

· (5), (6) ··· the local incentive constraints in the second period equilibrium.

· (22) ···the local incentive constraints in the first period symmetric equilibrium.

The lagrangean for the problem is written as follows.

$$L = \pi^*(\varphi, W) + \mu_1 \cdot \Delta_1 + \mu_2 \cdot V_{2L}{}^*$$

with the additional constraints, $\frac{1}{2} \leq \varphi \leq 1$ and $W \geq 0$.

For the notational simplicity, we incorporate the local incentive constraints in the first and second period into the constrained objective function of the principal (that is, the Lagrangean). ($\pi^*(\varphi, W)$ is, as the formula (32) shows, the expected profit of the principal achieved on the equilibrium path, for which the equilibrium incentives in the first and second period are substituted.) And Δ_1 is *the slack* associated with the first period global incentive constraint, and it equals to the LHS minus the RHS of the formula (26). Now, let us consider the first order conditions for the optimum.

First, as for φ (the production allotment share), we get

$$\frac{\partial L}{\partial \varphi} = \delta(1-\alpha)Q\left\{2(1+t)\left\{\frac{\partial h^*}{\partial \varphi}\right\} + \left[(e_W{}^* - e_L{}^*) + \frac{\partial e_L{}^*}{\partial \varphi} + \varphi \cdot \frac{\partial(e_W{}^* - e_L{}^*)}{\partial \varphi}\right]\right\}$$

$$+ \mu_1 \cdot \left[\frac{\partial \Delta_1}{\partial \varphi}\right] + \mu_2 \cdot \left[\frac{\partial V_{2L}{}^*}{\partial \varphi}\right] \leq 0 \tag{33}$$

where μ_1 and μ_2 are the non negative Kuhn Tucker Multipliers associated with the ex ante and ex post IR constraints. As for (33), we can check that

$$\frac{\partial \Delta_1}{\partial \varphi} = \alpha\delta Q\left\{\frac{1}{2}(e_W{}^* + e_L{}^*) + (\overline{K} + h_L + th^*)\right\} > 0$$

Hence, as φ increases, the global incentive constraint for the first period competition tends to be *relaxed*. One of the reasons is the marginal increase of the expected prize $:\frac{1}{2}(\overline{\upsilon} - \underline{\upsilon})$ in the second period, and the other is the decrease of the equilibrium revenue when he would lose in the first period competition and would be, as a result, assigned to the allotment of $1\text{-}\varphi$. This implies that the penalty becomes severe. These two effects induce the agents to participate in the first period competition or weaken their incentives to deviate (cheat) to the level of h_L.

Also, we see that

$$\frac{\partial V_{2L}{}^*}{\partial \varphi} = -\alpha Q\left\{\overline{K} + (1+t)h^* + e_L{}^*\right\} < 0$$

Hence, as φ increases, the global incentive constraint of the first period loser for the second period competition tends to be *tight*. In other words, the loser tends to exit (quit) from the ex post competition. Last, the effect of the increase of φ upon the equilibrium surplus of the principal is,

$$\frac{\partial \pi^*}{\partial \varphi} = \delta(1-\alpha)Q\left\{2(1+t)\left\{\frac{\partial h^*}{\partial \varphi}\right\} + \left[(e_W^* - e_L^*) + \frac{\partial e_L^*}{\partial \varphi} + \varphi \cdot \frac{\partial (e_W^* - e_L^*)}{\partial \varphi}\right]\right\}. \tag{33'}$$

The first blacket of (33') $2(1+t)\left\{\frac{\partial h^*}{\partial \varphi}\right\}$ is the effect which enhances the accumulated quality through the increase of the *ex ante* incentives by the increase of φ. Differentiating F.O.C (22) and arranging it, we get

$$\frac{\partial h^*}{\partial \varphi} = \frac{-\left(\frac{\partial \Delta \bar{\upsilon}}{\partial \varphi}\right) \cdot f(0)}{\{\delta(2\varphi - 1)\alpha Q(1+t)f(0) - g''(h^*)\}}$$

The denominator is negative because we assume that the local second order condition with respect to the first period investment incentives can be satisfied. Thus, the sign of the RHS of the above equation depends upon the sign of the numerator $\left(\frac{\partial \Delta \upsilon}{\partial \varphi}\right)$. The value is $\alpha Q(e_w^* + e_L^*) > 0$, and so, $\left(\frac{\partial h^*}{\partial \varphi}\right)$ is unambiguously positive. In other words, this implies that, as φ increases, the value of the equilibrium path ,where the winning agent chooses the level of e_w^* and the losing agent chooses the level of e_L^*, is enhanced, and that the marginal increase of this prize has the positive indirect effect upon the first period incentive. The second blacket is the effect which leads to the increase of the ex post (second period) incentive. First, from the proposition 1, $e_w^* - e_L^* \geq 0$ *(the inequality holds when* $\varphi > \frac{1}{2}$*).*

Next, $\left(\frac{\partial e_L^*}{\partial \varphi}\right)$ and $\left(\frac{\partial (e_W^* - e_L^*)}{\partial \varphi}\right)$ are each unambiguously negative and positive.

Hence, the increase of φ implies the subsidy (tax) to the winner (loser), and so it has a positive effect upon the ex ante capital accumulation but a large negative effect upon the ex post incentive of the loser. The point is that the increase of φ discourages the loser largely in the ex post stage. In addition to it, you can notice that the term of the explicit cost doesn't appear in this equation. In the traditional tournaments, the principal needs to spend the monetary cost (prize) in order to induce the incentives, while it doesn't appear in (33). This provides a hint as to why the principal may use the *nonmonetary* incentive scheme, such as "the assignment of allotments".

This is clear from the first order condition about the monetary prize:[7] W. The first order (Kuhn-Tucker) condition for the optimum for W is,

$$\frac{\partial L}{\partial W} = \delta(1-\alpha)Q\left\{2(1+t)\cdot\left[\frac{\partial h^*}{\partial W}\right]+\left[\frac{\partial e_L^*}{\partial W}+\varphi\cdot\frac{\partial(e_W^*-e_L^*)}{\partial W}\right]\right\} + \mu_1\cdot\left[\frac{\partial \Delta_1}{\partial W}\right]+\mu_2\cdot\left[\frac{\partial V_{2L}^*}{\partial W}\right]-1\leq 0 \tag{34}$$

The first term in the first large blacket { } is the marginal benefit for the principal obtained from the first period through the marginal increase of W. The second small blacket $\left[\frac{\partial e_L^*}{\partial W}+\varphi\cdot\frac{\partial(e_W^*-e_L^*)}{\partial W}\right]$ in the first large blacket is the marginal benefit obtained from the second period through increasing W. Also, the increase of W accompanies the direct marginal cost equal to one. When we consider the implications of these first order conditions, the noticeable points are two-fold. The first point is on the first term of the first blacket { }.
Differentiating F.O.C (22) with respect to W and arranging it, we get

$$\frac{\partial h^*}{\partial W} = \frac{-(2\Phi^*-1)\cdot f(0)}{\{\delta\cdot(2\varphi-1)\cdot\alpha Q\cdot(1+t)\cdot f(0)-g''(h^*)\}}$$

Since the denominator is negative from the fact that the S.O.C are locally satisfied in the neighborhood of h*, the sign of the above equation depends upon the sign of the numerator. When $\varphi>\frac{1}{2}$, then $e_W^*>e_L^*$ and $2\Phi(\Delta K+e_W^*-e_L^*)-1>0$. Thus, the sign of the total effect is positive. By the way, the increase of W induces both incentives of the winner and the loser in the second period. However, it was shown from the proposition 2 that the spread of incentives declines because the loser provides more incentives than the winner in the second period. If so, the incentives of both agents in the first period seem to decrease, because the future prize (reward) which will be obtained through winning in the first period competition seems to become smaller, since the loser rechallenges the winner severely in the second period. However, noticing that e_W^* and e_L^* are chosen optimally in the Nash equilibrium in

[7] We can interpret this prize as a kind of efficiency wages for inducing the dynamic incentives.

the second period, the total effect on the second period prize through the marginal change of $e_W{}^*$ and $e_L{}^*$ disappears from the *envelop theorem*. On the other hand, the direct effect of increasing efficiency wages. $\frac{\partial \Delta V}{\partial W} = 2\Phi(\Delta K + e_W{}^* - e_L{}^*) - 1 > 0$ remains. Hence, there remains the effect that the incentives in the first period are induced through the increase of the second period prize by increasing W. From (34), we see that the marginal increase of W costs one unit explicitly. This implies that even though the increase of W induces the investment incentives, it is costly in terms of the incentive cost. Also, we can check easily

$$\frac{\partial \Delta_1}{\partial W} > 0 \text{ and } \frac{\partial V_{2L}{}^*}{\partial W} > 0.$$

These inequalities show that if the principal increases W, the global incentive constraints tend to be *relaxed* or satisfied in the first period and the second period equilibrium, and this fact intuitively has a very reasonable implication that the increase of W enhances the fascination of the competition directly, and so makes the agents willing to participate in it.

In general, the expression of the Lagrangean is not concave in φ and W, since the responsiveness of the incentives to φ and W is involved, in particular, the global incentive constraints are not generally convex. So, the characterization of the optimal φ and W are involved, and it is difficult to fully characterize the overall solution. [$0 \le W$ and $\frac{1}{2} \le \varphi \le 1$ ($\varphi < \frac{1}{2}$ is not optimal for the principal. It is strictly dominated by $\varphi = \frac{1}{2}$)]. The problem for the principal is to maximize the Lagrangean (constrained objective function) subject to the two inequality constraints noticing the results of table 1 .Considering the implications of the first order conditions with respect to $e_W{}^*$, $e_L{}^*$, h^* and φ, W, we obtain the following propositions on the partial characterization of the optimal allotment (production share or quantity share ordered from the principal) and the second period prize (monetary payment).

Table 1 around here

[Proposition 6]

When the principal decreases φ (the allotment to the winner in the first period) marginally from 1, if the effect which generates the ex post competition (the increase of the average investment level) is larger than the decrease of the incentive of the first period and the increase of the *virtual cost* which arises in the control of the dynamic competition (the increase of the cost of the global incentive constraint), then $\varphi = 1$ (a corner solution viewed as "Elimination Tournament (separate the loser)") is not optimal from the viewpoint of the principal.

This is due to the following reasons. If the principal commits herself to $\varphi = 1$ in the ex ante stage, the situation of the ex post stage is nothing but a monopoly consisting of the winner, and so, the marginal incentive characterized by $C'(e^s) = \alpha Q$ arises. On the other hand, the marginal decrease (perturbation) from $\varphi = 1$ to $\varphi = 1-\xi$ leads to the discrete (1-order) loss of the first period incentive through the discrete change of the second period prize from the monopoly rent to the duopoly rent. But, as for the ex post stage, the decrease from $\varphi = 1$ to $\varphi = 1-\xi$ brings about the discrete increase of the ex post incentives, which is in the first order. Concretely, evaluating the latter part of the first blacket { } of the equation (33) at $\varphi = 1$, which represents marginal change of the ex post equilibrium incentives by the marginal change of φ, we get

$$\alpha Q + \left[\frac{\partial e_W^*}{\partial \varphi}\right]_{\varphi=1} = \alpha Q + \alpha Q \cdot \left((\phi' \cdot W)^2 + (\phi' \cdot W) + 1\right) > \alpha Q = C'(e^S) \tag{35}$$

(Here, we assumed the specific cost function $C(e) = \frac{1}{2}e^2$ in order to get the clear result. We should investigate the more general cost functions, but it is expected that the essence will be the same as above.) We obtain (35) from the computation of the comparative statics. Noticing that the second term is strictly positive, if we reduce the value of φ from 1 to $1-\xi$, <u>the first order gain, called "competition generating effect"</u>, comes about with respect to the ex post capital accumulation. Next, $\mu_1 \cdot \left[\frac{\partial \Delta_1}{\partial \varphi}\right]$ represents the enhancement of her profit through the relaxation of the ex ante global incentive constraints in the first period, by increasing φ.

On the other hand, $\mu_2 \cdot \left[\frac{\partial V_{2L}^*}{\partial \varphi}\right]$ represents the reduction of her profit through the tightness of the ex post global incentive constraint, which implies that it tends to be difficult for the loser to rechallenge the competition, and so, it becomes difficult or virtually costly for the principal to induce the loser to participate in the ex post competition.

The sum of these two terms: $\mu_1\left[\frac{\partial\Delta_1}{\partial\varphi}\right]+\mu_2\left[\frac{\partial V_{2L}*}{\partial\varphi}\right]$ implies the increase of "the incentive costs or the increase of the cost of controlling competition". These additional terms are *the total virtual costs* in controlling competition. This shows how the principal should pay (impose) the reward (penalty) to use the allotment φ as the incentive instrument. Since the negative effect upon the ex ante incentives is discretely large as φ moves from 1 to $1-\xi$, when "the competition creating effect in the second period" which dominates over the above negative incentive effect, is less than the increase of the incentive cost of controlling competition evaluated at $\varphi=1$, the principal should reduce φ from 1, establishing the ex post competition for the payment W in order to increase her profit. This means that nonelimination tournaments, letting the loser rechallenge in the 2-period, are preferable, in terms of the total balance of the ex ante and ex post effects, especially under the situations where the second period capital accumulations are relatively vital.

This proposition 6 tends to be satisfied as the rate of technology transfer : t is close to 1. It is because, by the fact that the equilibrium incentives *ex post* increases as t is close to 1 (Corollary 2), and also the ex post incentive constraint tends to be satisfied, the virtual expected profit from the second period competition is increased from the viewpoint of the principal. In this case, it doesn't contradict with the intuition that $\varphi=1$ (the principal gives the monopoly rent to the winner) is not optimal.

Next,

[Proposition 7]

If the uncertainty at the end of the first period is sufficiently small, and if the bargaining power α of agents and the market demand Q is sufficiently large, $\varphi=\frac{1}{2}$ is not optimal, in other words, adopting an allotment scheme $\varphi>\frac{1}{2}$ is optimal, or *"endogenously" chosen* by the principal.

The economic intuition of this proposition is the same as the proposition 6. When the principal commits herself to $\varphi=\frac{1}{2}$, the equilibrium incentive in the second period is characterized by the marginal condition

$$\frac{\alpha Q}{2}+\phi(0)\cdot W=C'(e^*) \tag{36}$$

On the other hand, as for the investment decision in the first period, the two agents simultaneously and independently decide how much to invest, expecting to obtain the expected rent $[2\ \Phi(K_{i2}\text{-}K_{j2})\text{-}1]\cdot W$, given the difference of capital stocks $K_{i2}\text{-}K_{j2},\ i\neq j$ at the start of the second period subgames. Agent i solves the following problem, given the incentive of agent j ; h_j.

$$\underset{\{h_i\}}{Max}\ \delta\cdot\frac{\alpha Q}{2}\left[\overline{K}+h_i+e^*\right]+\delta\cdot F(h_i-h_j)\cdot\left[2\Phi(K_{i2}-K_{j2})-1\right]\cdot W-g(h_i)$$
$$i\neq j, i=1,2$$

The first order condition with respect to h_i is,

$$\delta\cdot\frac{\alpha Q}{2}+\delta f(h_i-h_j)\cdot\left[2\Phi(K_{i2}-K_{j2})-1\right]\cdot W+\delta\cdot F(h_i-h_j)\cdot\left[2(1-t)\phi(K_{i2}-K_{j2})\right]\cdot W=g'(hi)$$

$i\neq j,\ i=1,\ 2$

The first term is the share α of the marginal increase of the quality, under the expected allotment of $\frac{1}{2}$Q .The second term is the increase of the expected rent in the second period through the marginal improvement of the winning probability in the first period, and the third term is the marginal increase of the expected rent in the second period through the marginal improvement of the winning probability in the second period. Now, we restrict our restriction to the symmetric equilibrium in the first period. Then, we obtain the following equation, characterizing the equilibrium investment level.

$$\delta\frac{\alpha Q}{2}+\delta\cdot(1-t)\cdot\phi(0)\cdot W=g'(h^*) \tag{37}$$

If the agents play the "two stage" investment game under the regime of $\left\{\varphi=\frac{1}{2}\ \text{and}\ W>0,\right\}$ the equilibrium investment levels are characterized by both (36) and (37). In the left hand of both equations, the first term corresponds to *the direct effect* such that the agents expect to get the allotment of $\frac{Q}{2}$ in equilibrium and receive the share of α of one unit increase of the capital accumulation by their investments, as the private revenues. The second term is called *the (marginal) strategic effect,* which is the product of the monetary payment and the marginal improvement of the probability of obtaining W. The sum of these two corresponds to the marginal benefit of the investment in each period. Note that in (37), if $t=1$, the strategic effect does disappear, because his performance (accumulation) is perfectly transferred to the rival, so that he is perfectly exploited by his rival the progress obtained through his costly effort. Next, we consider the marginal change of φ *from* $\frac{1}{2}$ *to* $\frac{1}{2}+\xi$. Then, the incentive loss in the second period is only in the second order through the marginal change of the equilibrium level. While, as for the incentives in the first period, the discrete prize can be obtained if the own performance be over the rival's and so, in the reward function, *the discontinuous jump* arises at the achieved level of the rival's capital accumulation. (See Figure 3.)

The essential point is as follows. When $\varphi=\frac{1}{2}$, the marginal strategic effect arises through the observability of the progress on the way at the end of the first period. On the other hand, when $\varphi=\frac{1}{2}+\xi$, where ξ is small, *the discrete tournament effect* emerges as for the first period incentives through the discrete prize (16), which increases the first period equilibrium incentives discretely. Hence, the principal can induce the more incentives in terms of the intertemporal incentives by perturbing the allotment from $\varphi=\frac{1}{2}$ *to* $\frac{1}{2}+\xi$. Technically, this is sufficient if the condition that

$$\xi \geq \frac{1}{[\alpha Q]\cdot[f(0)]} \tag{38}$$

is satisfied[8]. This is a sufficient condition of the optimality of adopting the two stage tournaments (the allotment and the monetary prize). (38) tends to be satisfied, as f(0) is larger, in other words, the uncertainty is smaller. Let us mention the above argument in the viewpoint of *the incentive cost.* We

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[8] Readers can request the detail of the derivation of this sufficient condition from the author. He is investigating other sufficient conditions and a method, which depicts the structure of the overall game more clearly.

can say that the principal can increase the equilibrium profit, by adopting the allotment scheme because he can reduce the incentive cost (the implementation cost, in this case, the monetary reward or premium) to induce the constant level of incentives on the equilibrium path(9).

Next, we obtain the following proposition about monetary prize (efficiency wage) W.

[Proposition 8]

It is optimal to set the positive monetary prize : W in the final period under the nonelimination tournaments *(i.e,* $\varphi > \frac{1}{2}$ *and* $W>0$*).*

As we verified before, $\frac{\partial \Delta V}{\partial W} = 2\Phi(\Delta K + e_W{}^* - e_L{}^*) - 1 > 0.$

So, the intuition of this proposition is clear. If the principal increases W under the condition of $\frac{\partial \Delta V}{\partial W} > 0$, it leads not only to the increase of the investment levels $(e_W{}^*,\ e_L{}^*)$ in the second period, but also the increase of the equilibrium investment level h^*in the first period through the increase of the prize Δ V, as the payoff spread in the asymmetric equilibrium generated in the second period. Hence, the prize W set for the final winner has the long term effect, and it is a device, combined with the allotment scheme, for inducing the incentives from agents over two periods.

4 Discussions

4.1 Sequential assignment of allotments

If the principal adopts the sequential assignment of allotments, he may be able to increase his equilibrium profit. The essential logic is as follows. If the principal can divide the first period into two stages, and assign φ to the winner in the first substage and $\frac{1}{2}-\varphi$ to the loser, where $\varphi > \frac{1}{4}$, the first order increase of the incentive will be generated in the first substage while the second order decrease of the incentive will be generated in the second substage in the first period. If the net increase of intertemporal incentives in the first period be larger than the marginal cost increase of the incentive in the first substage, the net positive surplus be generated. The principal can transfer the gain from relaxing the ex ante IR constraint into the second period, and reduce the base payment for the loser to participate in the ex post competition discretely, keeping the prize W constant. Hence, the profit of the principal will be increased.

This can be stated that the principal can decrease the reward (premium) W in order to induce a given level of intertemporal incentives. We may be able to find the optimal number of assignment. But, the essential logic is as above.

4.2 Comparative Statics:the effects of the exogenous parameters upon the set of the credible contracts and the optimal incentive schemes

(1) The bargaining power α of the agents :this is a 'zero-sum' factor between the principal and the agents.

As α becomes smaller, the participation constraints for the agents becomes tighter. Hence, the principal must increase W for each φ,in order to increase the fascination of the competition. The effect upon the payoff of the principal is ambiguous, because the bargaining power $1-\alpha$ increases but the monetary payment W also increases and the induced incentives on the equilibrium path may increase or decrease.

(2) The demand (total trade volume) Q: this is a 'non-zero- sum' factor among the principal and the agents. The interesting case is the case when Q decreases. As it decreases, the incentive levels themselves decreases. But ,from the F.O.Cs (22) ,the competitive pressure for the allotment tends to push the incentives of the agents over the hold up levels h_s.This suggests that this kind of procurement mechanism with supplier competition tends to be favorable over the one with a single supplier ,when the market is slump .In practice, the Japanese subcontracting system with supplier competition (' Controlled competition ') has rather established after the experiences of the oil shock.

(3) The technology transfer rate t: different from Anton-Yao(1988)and Riordan-Sappington(1989), let us investigate <u>how t affects upon the set of credible contracts</u> ,especially, the global incentive constraint of the loser in the second period. Needless to say, as t increases, not only the ex post equilibrium incentives will increase ,but also the global incentive constraint of the loser will be relaxed. Hence, the expected profit of the principal will increase in the viewpoint of the second period. Next, we shall investigate the effect upon the first period .Noticing that the technology transfer is made <u>after</u> the interim ranking with the allotment, the technology transfer will increase the first period prize from (16) and so, the first period equilibrium incentives. Thus, the equilibrium profit of the principal in the first

period will also increase.

4.3 Sequential Rationality constraints of the principal for φ and W

This increase the number of the constraints that the principal must satisfy, and so, will reduce the attainable equilibrium profit for the principal.

4.4 The relation specificity and the effect of the outside option.

The intuition is that as the relation specificity is reduced, the outside option increases and it becomes harder to let the loser participate in the ex post competition. Hence, the principal must compensate the loser for the participation in the form of more wage payment.

4.5 The more flexible allotment scheme: $\Psi(\Delta K)$

This will clearly increase the equilibrium payoff of the principal, because the number of the instruments controlled by her increases.

4.6 The Renegotiation Problem by the Principal in the interim stage.

Suppose that at the interim stage after the first period competition, the principal can offer the renegotiation of the initial contract. Then, she must satisfy the individual rationality constraints for both agents. Especially, slack can be generated from the global incentive constraints for all types except for the worst type $\underline{\varepsilon}$, keeping the size of prize constant. However, such type of renegotiation connects the reduction of the principal's base payment (hence, the total payment) in the second period to the tightness of the ex-ante individual rationality constraints of the agents .The reduction of the base payment is "punished" by being required to relax the tightened individual rationality constraint. This gives an insight that the renegotiation can redistribute the payoff contingent upon the states of nature after the first period, by exploiting the slackness in the agents' ex post participation constraint ,but that this requires to the principal other forms of compensations .[9]If the principal combines the sequential assignment of

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[9] This idea that *slack can be traded* through a renegotiation has an analogy with the recent financial contracting literatures ,e.g.Webb (1992).

allotments ,generating the surplus in the first period ,with the ex post renegotiation mentioned above , she will be able to increase the expected profit by exploiting the slack in the second period : $\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} V_{2L}\left(\overline{K},h^{*},\varepsilon\right)f(\varepsilon)d\varepsilon$, satisfying the ex-ante individual rationality constraint.

5. Concluding Remarks

In this paper, we modeled a situation where a few ,fixed members (e.g. part suppliers) compete dynamically in organizations and investigated how a limited , but active intervention policy by the principal into the competition affects the dynamic incentives of the agents and can induce the higher capital accumulation. In Japanese style of competitions, it is often said that there does not exist the loser in the true sense, in its sense, nonelimination tournaments. When the principal cannot explicitly write and commit herself to the state-contingent initial contract, there often occurs the 'Hold- up problem ',leading to the underinvestment problems. Even in such situations , the principal can design the multi-dimensional races, and induce the first period incentives ,mainly seeking for the nonmonetary reward (larger allotment) ,and at the same time, keeping the interim equality in capital accumulation levels through the technology transfer , induce the active competition even in the second period for the other prize. This type of competition can be interpreted as the voice mechanism suggested by A.Hirshman (1970) ,especially, the voice mechanism designed by the principal ,with monetary and nonmonetary incentive schemes.

Next, in the solutions of $\varphi = \frac{1}{2}$,under the information structure where the relative performance at the interim stage is *observable* with each other , the subgame played by the agents is essentially two stage game .In this case, as the second term of (37) shows , the ex- ante incentives among the agents has voluntarily increased through the marginal strategic effect. On the other hand, under the information structure corresponding to the *unobservability* of capital stocks at the interim stage in the solutions of$\varphi = \frac{1}{2}$,the two agents will play the one stage game, where they *commit themselves to the capital accumulation path over the two periods* ,and so the strategic effect: $(1-t)\cdot\phi(0)\cdot W$ does not occur. This can be stated that if the principal can manage the information structure such that the two agents can observe each other's capital accumulation outcome in the first period ,more incentives can be induced by $(1-t)\cdot\phi(0)\cdot W$ in the first period ,for a constant prize:W .Adding to it ,we investigated how the principal strategically utilizes the incentive creation effect through the change of the information structure, by adopting the allotment scheme. As a result, we showed that under some sufficient condition, the principal adopts the allotment scheme : $\varphi^* > \frac{1}{2}$ and can induce the larger incentive through the discrete prize as a difference of equilibrium payoffs ,and so for the intertemporal incentives. This type of information management policy and the principal's strategy of utilizing the change of the induced dynamic incentives is essential both theoretically and for an explanation of the growth of organizations.[10]

We didn't give a full characterization of the solution of the overall Kuhn-Tucker problem. When the exogenous conditions change, how the optimal solution (the combinations of the two strategies: φ and W) are affected in an inner solution? Under some exogenous conditions , a corner solution of $\varphi = \frac{1}{2}$ (not using the two stage tournaments) or $\{\varphi^* = 1$ and $W = 0\}$(elimination of the loser from the ex-post competition) may be chosen as an optimal solution of the Kuhn-Tucker problem. However, we obtained a result that when the market demand is large ,the uncertainty is small, and the spillover of the technology and knowledge tends to occur, two stage tournaments with allotment ($\frac{1}{2} < \varphi < 1$ and $W > 0$) tends to be optimally chosen. This condition commensurate with the situations under the catch-up economy or the Japanese subcontracting systems in the second half of 1980's.

We discussed some theoretical remarks in section 4. In the future, we will explore a theoretical analysis of multi-dimensional races ,including equilibrium concepts, control of races, and problems about commitment and renegotiation of the designer of races.

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[10] It is well known that a change of information structures brings about a change of strategy spaces ,leading to the change of equilibrium incentives .As for it, see Fudenberg -Tirole (1987) .From a practical point of view ,Itami (1988) points out how an information management affects the incentives of quality improvement in the Japanese subcontracting system.

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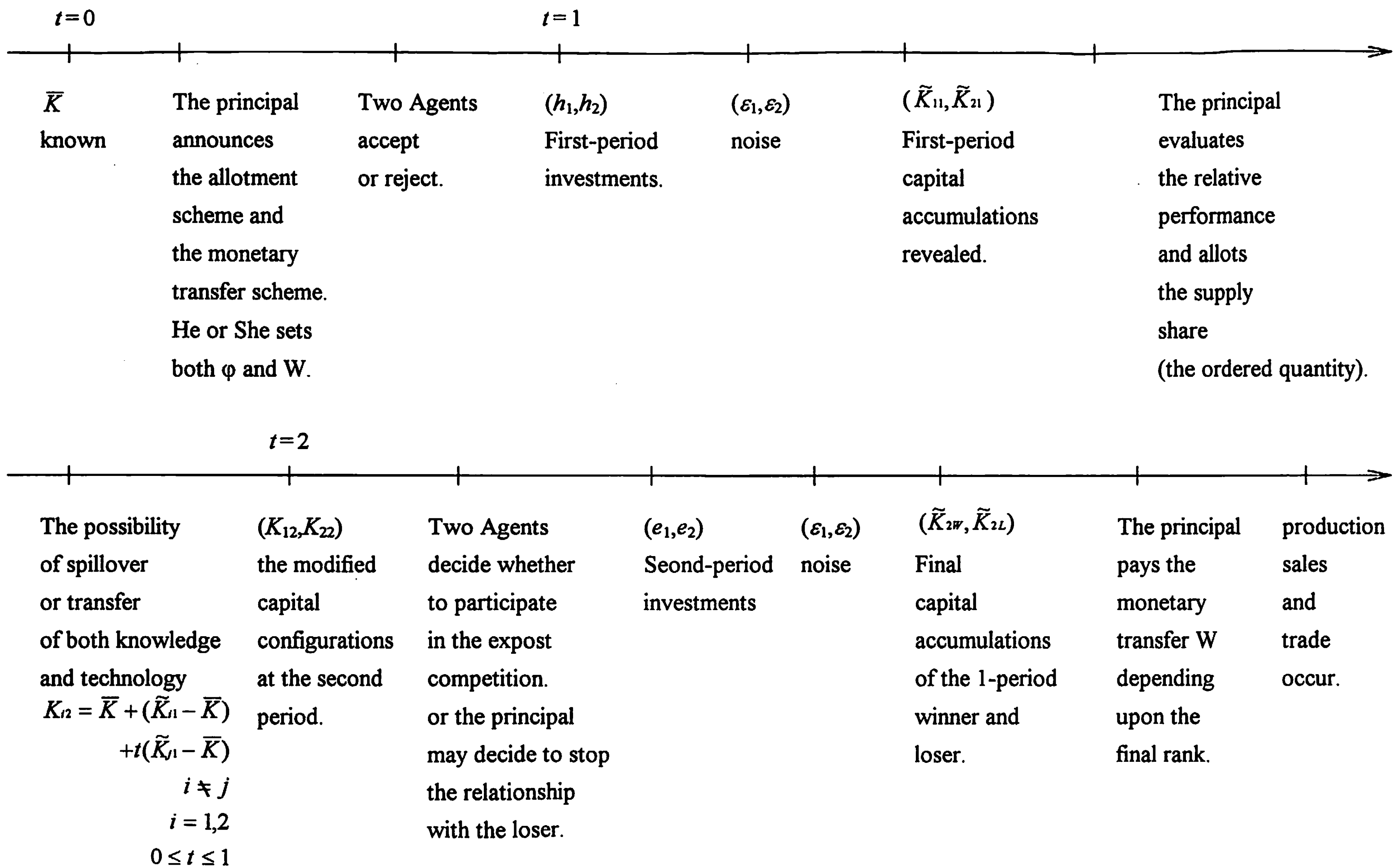
$t=0$
$\overline{K}$ known
The principal announces the allotment scheme and the monetary transfer scheme. He or She sets both φ and W.
Two Agents accept or reject.
$t=1$
(h_1,h_2) First-period investments.
$(\varepsilon_1,\varepsilon_2)$ noise
$(\tilde{K}_{11},\tilde{K}_{21})$ First-period capital accumulations revealed.
The principal evaluates the relative performance and allots the supply share (the ordered quantity).
The possibility of spillover or transfer of both knowledge and technology
$K_{i2}=\overline{K}+(\tilde{K}_{i1}-\overline{K})$
$+t(\tilde{K}_{j1}-\overline{K})$
$i \neq j$
$i=1,2$
$0\leq t\leq 1$
$t=2$
(K_{12},K_{22}) the modified capital configurations at the second period.
Two Agents decide whether to participate in the expost competition. or the principal may decide to stop the relationship with the loser.
(e_1,e_2) Seond-period investments
$(\varepsilon_1,\varepsilon_2)$ noise
$(\tilde{K}_{2W},\tilde{K}_{2L})$ Final capital accumulations of the 1-period winner and loser.
The principal pays the monetary transfer W depending upon the final rank.
production sales and trade occur.

Figure 1

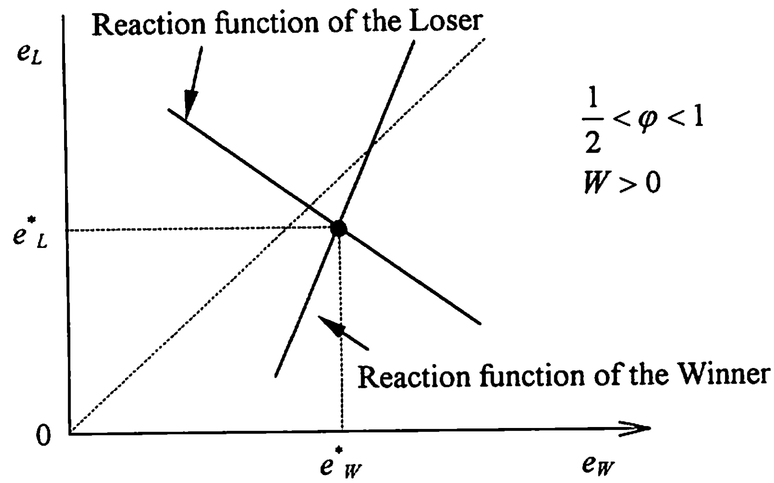

Figure 2.1

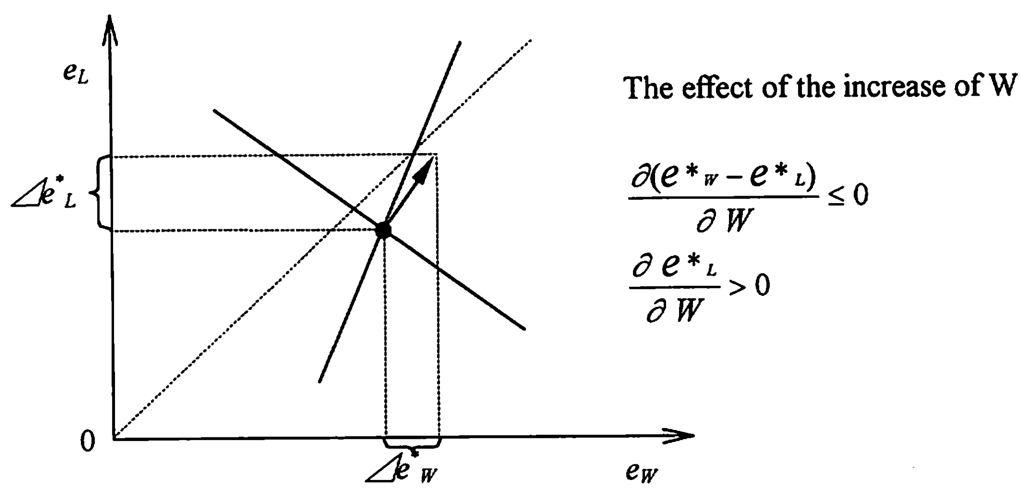

Figure 2.2

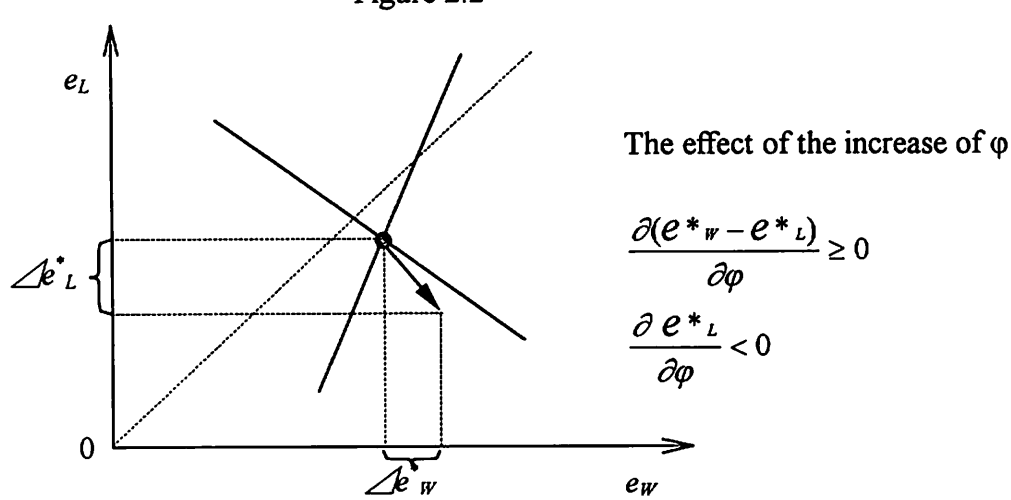

Figure 2.3

Figure 2・1～2・3 The Nash equilibrium in the second period and the effect of the increase of φ and W upon the equilibrium.

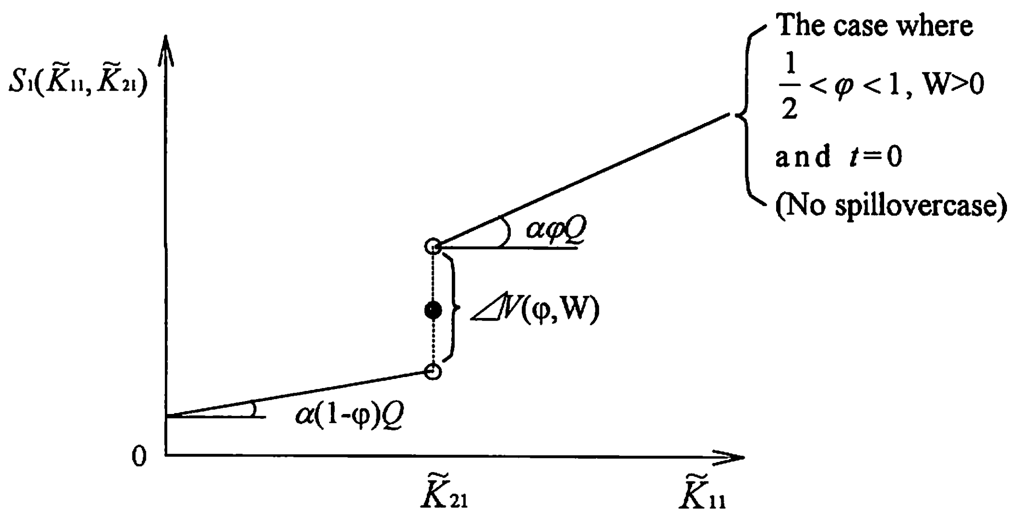

Figure 3 The Tournament Scheme with which the agents are faced in the first period (This scheme is "endogenously" chosen by the principal through the allotment scheme)

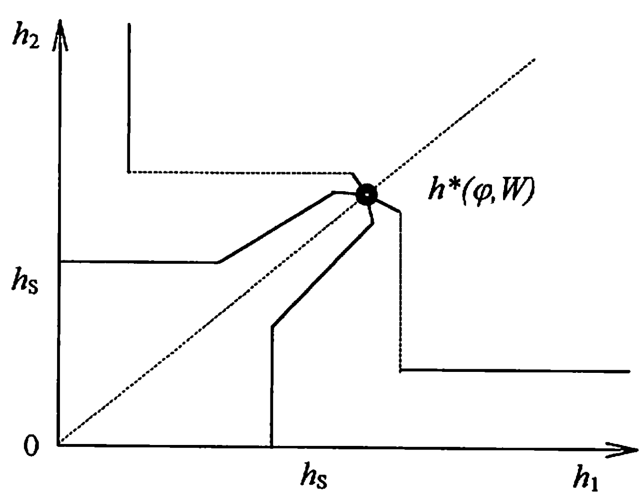

Figure 4 The equilibrium investment level in the first period : $h^*\ (\varphi, W)$

Table 1

two incentive devices / Benefit and cost	φ	W
The effect upon the ex ante incentives : $h^*(\varphi, W)$	+	+
The effect upon the weighted average of the ex post incentives : $\varphi \cdot e_W^* + (1-\varphi) \cdot e_L^*$	- The fall of the "competitive pressure" Figure 2.2	+ The revival of the "competitive pressure" Figure 2.3
The Direct Cost for the principal	0	1
The effect upon the ex ante global incentive constraint	+ **(relaxed)**	+ **(relaxed)**
The effect upon the ex post participation constraint of the laser	- **(tighten)**	+ **(relaxed)**

The effect upon both the ex ante and ex post equilibrium incentives brought about by the marginal increases of the two incentive devices, in the rang of $\frac{1}{2} \le \varphi < 1$ and W≥0).